

More References for “Type-2 Fuzzy Sets as Well as Computing With Words”: Organized by the Sections of that Article

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Notes

1. For completeness, the references that are given at the end of the article are repeated here as well, but here they are organized according to the sections of the article.
2. If the same reference is used in more than one section of the article it is restated for each section in which it is used, but, if a book, then a specific chapter or section is referred to.
3. Except for some papers that are mentioned in Section II. E, application papers are not included herein, but they are included for T2 FSs and fuzzy systems in “References for Type-2 Fuzzy Sets and Fuzzy Systems”.

I. INTRODUCTION

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II. WHY TYPE-2 FUZZY SETS SHOULD BE USED

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None

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G. Algorithms for TR

Note: Any TR algorithm that has been developed for IT2 FSs can also be used for GT2 FSs when the horizontal-slice representation is used for the latter.

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